Model Question Paper-I with effect from 2016-17

USN | | | | | | | | | | | **15MAT41**

Fourth Semester B.E.(CBCS) Examination
Engineering Mathematics-IV
(Common to all Branches)

Time: 3 Hrs   Max.Marks: 80

**Note: Answer any FIVE full questions, choosing at least ONE question from each module.
Statistical tables may be provided.**

Module-I

1. (a) Using Taylor's series method, solve the initial value problem $\frac{dy}{dx} = xy^2 - 1, y(0) = 1$ and hence find the the value of y at the point $x = 0.1$. **(05 Marks)**

   (b) Employ fourth order Runge - Kutta method to solve $\frac{dy}{dx} = (y^2 - x^2)/(y^2 + x^2), y(0) = 1$, at $x = 0.2$. (Take h =0.2) **(05 Marks)**

   (c) Using Adam-Bashforth predictor-corrector method to solve $\frac{dy}{dx} = x^2(1 + y)$ given that $y(1) = 1, y(1.1) = 1.2330, y(1.2) = 1.5480, \& \ y(1.3) = 1.9790$ to find $y(1.4)$. **(06 Marks)**

OR

2. (a) Using modified Euler's method find $y(0.1)$, given $\frac{dy}{dx} + y - x^2 = 0$ with $y(0) = 1$. Perform two iterations at each step, taking $h = 0.05$. **(05 Marks)**

   (b) Use fourth order Runge - Kutta method to find $y(1.1)$, given $\frac{dy}{dx} = xy^{1/3}, y(1) = 1$. (Take h =0.1) **(05 Marks)**

   (c) Apply Milne's predictor-corrector formulae to compute $y(0.8)$, given **(06 Marks)**

   $\frac{dy}{dx} = x - y^2$ and

x	0	0.2	0.4	0.6
y	0	0.0288	0.0788	0.1799

Module-II

3. (a) Given $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1, y(0) = 1, y'(0) = 0,$ evaluate $y(0.1)$ using Runge - Kutta method. **(05 Marks)**

   (b) Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials. **(05 Marks)**

   (c) Solve the Bessel's differential equation viz., $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ to write the complete solution in terms of $J_n(x)$. **(06 Marks)**

OR

4. (a) Apply Milne's method to compute $y(0.8)$ given that $\frac{d^2y}{dx^2}=1-2y\frac{dy}{dx}$ and the following table of initial values: **(05 Marks)**

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

(b) With usual notation, prove that $J_{1/2}(x)=\sqrt{(2/\pi x)}\sin x$. **(05 Marks)**

(c) c) If $x^3+2x^2-x+1=aP_0(x)+bP_1(x)+cP_2(x)+dP_3(x)$, find the values of a,b,c,d **(06 Marks)**

Module-III

5. (a) Derive Cauchy-Riemann equation in polar form. **(05 Marks)**

(b) Using Cauchy's residue theorem to evaluate the integral $\int_C \frac{z^2dz}{(z-1)^2(z+2)}$ where C is the circle $|z|=3$ **(05 Marks)**

(c) Find the bilinear transformation that transforms the points $z_1=i, z_2=1, z_3=-1$ on to the points $w_1=1, w_2=0, w_3=\infty$ respectively. **(06 Marks)**

OR

6. (a) State and prove Cauchy's integral formula. **(05 Marks)**

(b) Given $u-v=(x-y)(x^2+4xy+y^2)$, find the analytic function $f(z)=u+iv$ **(05 Marks)**

(c) Discuss the transformation $w=z+(1/z), z\neq 0$. **(06 Marks)**

Module-IV

7. (a) Derive mean and standard deviation of the binomial distribution. **(05 Marks)**

(b) In a certain factory turning out razor blades, there is a small chance of 0.002 for a blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing i) no defective ii) one defective iii) two defective blades, in a consignment of 10,000 packets. **(05 Marks)**

(c) The joint probability distribution for two random variables X and Y is given below:

X \ Y	-2	-1	4	5
1	0.1	0.2	0.0	0.3
2	0.2	0.1	0.1	0.0

Determine (i) marginal distribution of X and Y (ii) covariance of X and Y (iii) correlation between X and Y **(06 Marks)**

OR

8.(a) The average daily turn out in a medical store is Rs. 10,000 and the net profit is 8%. If the turn out has an exponential distribution , find the probability that the net profit will exceed Rs. 3000 each on two consecutive days. **(05 Marks)**

(b) The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be (i) less than 65 (ii) more than 75 (iii) between 65 and 75, given $\Phi(1) = 0.3413$. **(05 Marks)**

(c) The Joint distribution of two random variables *X* and *Y* is as follows:

X \ *Y*	-4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Compute (i) *E(X)* and *E(Y)* (ii) *E(XY)* (iii) *Cov (X,Y)* &
(iv) $\rho(X,Y)$ **(06 Marks)**

Module-V

9. (a) Explain the terms:(i)Null hypothesis (ii)Confidence intervals (iii)Type I and Type II errors **(05marks)**

(b) A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5,3,8,-1,3,0,6,-2,1,5,0,4. Can it be concluded that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 d.f is 2.201). **(05 marks)**

(c) Show that the Markov chain whose transition probability matrix $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is irreducible. Also, find the corresponding stationary probability vector. **(06 marks)**

OR

10. (a) In an elementary school examination the mean grade of 32 boys was 72 with a standard deviation of 8 while the mean grade of 36 girls was 75 with a standard deviation of 6. Test the hypothesis that the performance of girls is better than boys. **(05 marks)**

(b) Four coins are tossed 100 times and the following results were obtained.

Number of Heads	0	1	2	3	4
Frequency	5	29	36	25	5

Fit a binomial distribution for the data and test the goodness of fit ($\chi^2_{0.05} = 9.49$ for 4 d.f.). **(05marks)**

(c) Every year, a man trades for his car for a new car. If he has Maruti, he trade it for a Ford. If he has a Ford, he trade it for a Hyundai. However, if he has a Hyundai, he is just as likely to trade it for a new Hyundai as to trade it for a Maruti or a Ford. In 2014, he bought his first car which was a Hyundai. Find the probability that he has (a) 2016 Ford (b) 2016 Hyundai (c) 2016 Maruti. **(06 marks)**

*****